A Local-Search Algorithm for Steiner Forest

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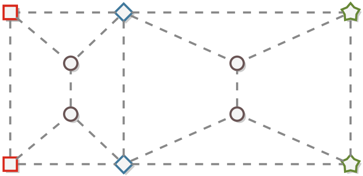
The Steiner Forest Problem

Input

Graph	$G = (V, E)$
Terminal pairs	$(s_1, \bar{s}_1), \ldots, (s_k, \bar{s}_k) \in V \times V$
Edge costs	$c : E \to \mathbb{R}^+$

Output

Minimum cost forest $F \subseteq E$ containing s_i-$\bar{s}_i$-path for all $i = 1, \ldots, k$



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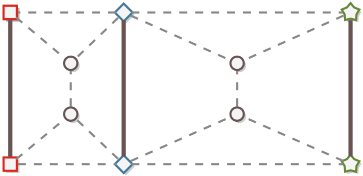

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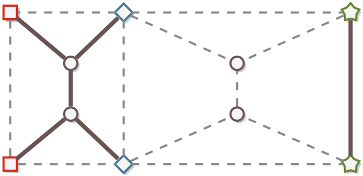


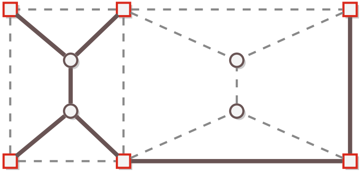
The Steiner Tree Problem

Input

Graph	$G = (V, E)$
Terminals	$s_1, \ldots, s_k \in V$
Edge costs	$c : E \to \mathbb{R}^+$

Output

Minimum cost tree $T \subseteq E$ containing all s_i



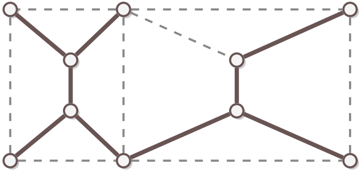
The Minimum Spanning Tree Problem

Input

Graph	$G = (V, E)$
Terminals	V
Edge costs	$c : E \to \mathbb{R}^+$

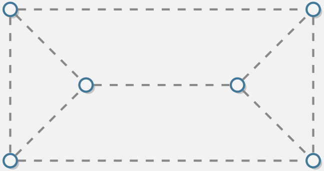
Output

Minimum cost tree $T \subseteq E$ containing all v



Local Search

Example: Local search for MST with euclidean distances



Local search algorithm.

1. Start from arbitrary feasible solution.
2. Reach next feasible solution by executing single edge swaps.
3. Iterate until no improving swap $\rightsquigarrow$ Local optimum reached.

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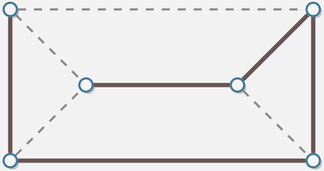

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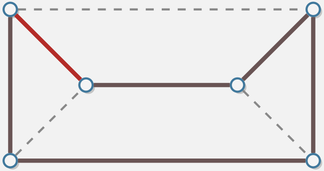

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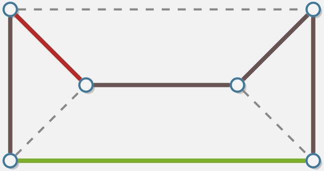

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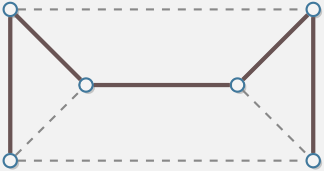

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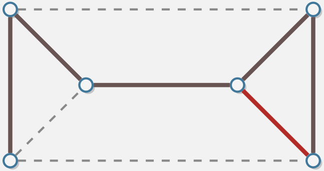

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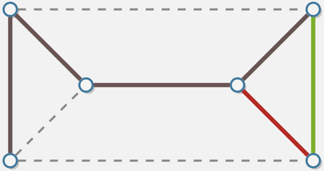

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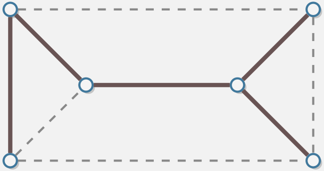

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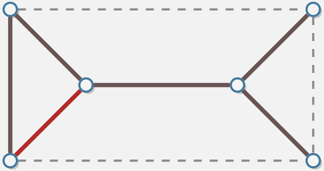

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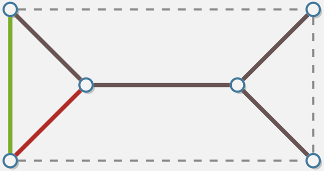

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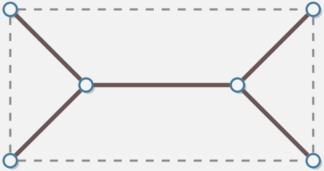

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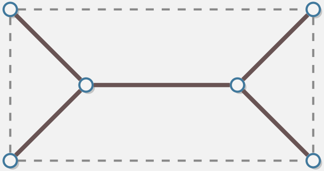

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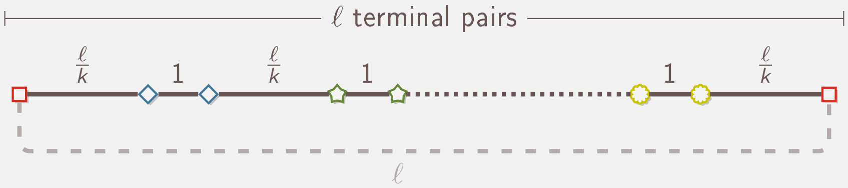
- For MST, this is optimal!
- $\rightsquigarrow$ 2-approximation for Steiner Tree

Local Search for Steiner Forest: Simple moves don't work

Choose ℓ and k with $\ell \gg k$



Need to remove more than $O(1)$ edges.

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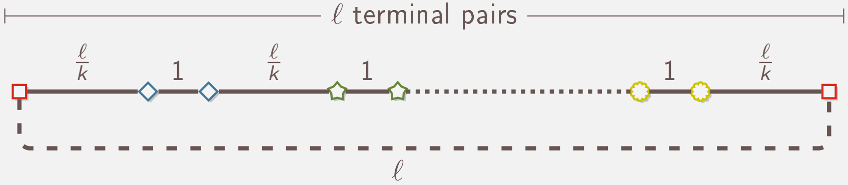

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- Insert edge of cost ℓ
- Unless more than k edges are removed, no improving move.
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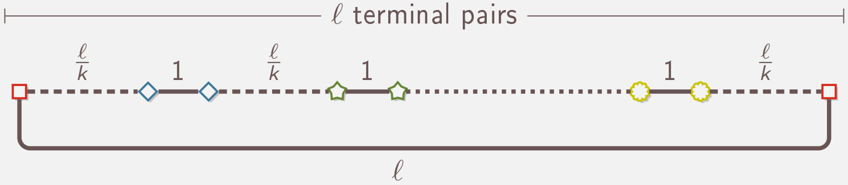

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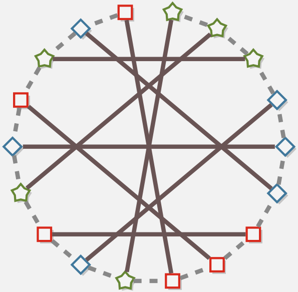

- Solid edges cost 4, dashed edges cost 1.

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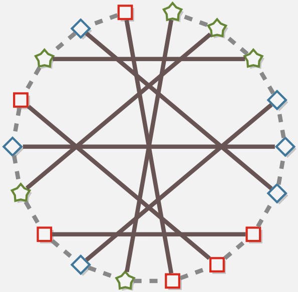

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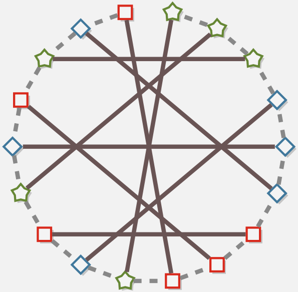

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- More general version of this example: $\Omega(\log n)$ approximation factor.

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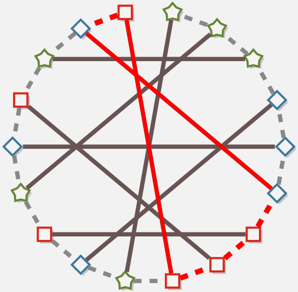

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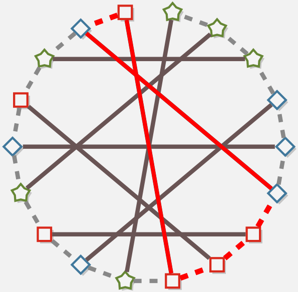

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Solution: Complex puzzle with a potential function and involved local steps.

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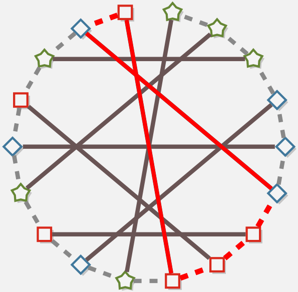

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Theorem

There is a non-oblivious local search algorithm for the Steiner Forest Problem with a constant locality gap.